NAG Toolbox for MATLAB

f07jn

1 Purpose

f07jn computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n Hermitian positive-definite tridiagonal matrix, and X and B are n by r matrices.

2 Syntax

```
[d, e, b, info] = f07jn(d, e, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07jn factors A as $A = LDL^{\mathrm{H}}$. The factored form of A is then used to solve the system of equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **d**($*$) – **double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

The n diagonal elements of the tridiagonal matrix A.

2: **e**($*$) – **complex array**

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

The $(n - 1)$ subdiagonal elements of the tridiagonal matrix A.

3: **b**(**ldb**,$*$) – **complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The dimension of the array **d**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: **nrhs_p** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters

1: **d**$(*)$ **– double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

The n diagonal elements of the diagonal matrix D from the factorization $A = LDL^{\mathrm{H}}$.

2: **e**$(*)$ **– complex array**

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

The $(n-1)$ subdiagonal elements of the unit bidiagonal factor L from the LDL^{H} factorization of A. **e** can also be regarded as the superdiagonal of the unit bidiagonal factor U from the $U^{\mathrm{H}}DU$ factorization of A.

3: **b**(**ldb**,$*$) **– complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

If **info** $= 0$, the n by r solution matrix X.

4: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **nrhs_p**, 3: **d**, 4: **e**, 5: **b**, 6: **ldb**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the leading minor of order i is not positive-definite, and the solution has not been computed. The factorization has not been completed unless $i = \mathbf{n}$.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

f07jp is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, f04cg solves $Ax = b$ and returns a forward error bound and condition estimate. f04cg calls f07jn to solve the equations.

8 Further Comments

The number of floating-point operations required for the factorization of A is proportional to n, and the number of floating-point operations required for the solution of the equations is proportional to nr, where r is the number of right-hand sides.

The real analogue of this function is f07ja.

9 Example

```
d = [16;
     41;
     46;
     21];
e = [complex(16, +16);
     complex(18, -9);
     complex(1, -4)];
b = [complex(64, +16);
     complex(93, +62);
     complex(78, -80);
     complex(14, -27)];
[dOut, eOut, bOut, info] = f07jn(d, e, b)
```

```
dOut =
    16
     9
     1
     4
eOut =
   1.0000 + 1.0000i
   2.0000 - 1.0000i
   1.0000 - 4.0000i
bOut =
   2.0000 + 1.0000i
   1.0000 + 1.0000i
   1.0000 - 2.0000i
   1.0000 - 1.0000i
info =
           0
```